

Research on Structural Topology Optimization Methods and Application

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Abstract. Topology optimization is one of the most important methods of structure optimization, which is to optimize the weight and performance of the structure by changing the distribution of materials. Firstly, this paper introduces five main methods of topology optimization: homogenization method, variable density method, progressive structure optimization method, independent continuous mapping method and level set method. Then, the development process, optimization principle and effect of each method in practical application are elaborated. Based on the theoretical basis of these methods, the advantages and limitations of these methods in different situations and practical applications are analyzed and compared. In addition, the applicability of these methods in different scenarios is summarized, and their effects and limitations in solving specific engineering problems are pointed out. Finally, the future research direction of topology optimization and some potential improvement measures are proposed to find better solutions in complex engineering problems. The research results show that each topology optimization method has its own advantages and disadvantages. Each method is suitable for different application scenarios, so it is necessary to choose the appropriate method according to the specific needs.

Keywords: Topology optimization; Structure optimization method; Variable density method.

1. Introduction

The development of human society cannot be separated from complex structural systems, and structural systems cannot be separated from structural optimization design supported by physical theory. Structural optimization is an optimized structure based on mathematical theory and computer simulation under given constraints. Structural optimization is mainly divided into three levels: size optimization, shape optimization and topology optimization [1]. Size optimization is to adjust the cross-sectional area of the rod to the most appropriate size. Shape optimization optimizes the structural performance by changing the node positions of the rod structure. Topology optimization improves the structure by changing the node distribution of the rod structure, the rod connection relationships between nodes, or the number and position of continuum openings. To achieve structural lightweight performance, topology optimization helps in finding the optimal material distribution within a given design space. It operates under specified load and volume constraints. This approach is ideal for applications that require high performance and lightweight designs, enabling precise adjustments to meet strict performance and weight criteria. In the past few decades, topology optimization has had many theoretical innovations in various scientific aspects, even in some manufacturing industries. With the continuous in-depth research on topology optimization methods, a complete theory and calculation method has been designed to meet the design needs of most industries. It has been widely used in construction, aviation, machinery, marine engineering and other fields [1].

The purpose of this article is to discuss the development direction and future trends of topology optimization, point out the mainstream methods and application directions, and summarize and link them. Give suggestions and evaluations on the application of these topology optimizations in different fields, and provide opinions and prospects for their future development.

2. Structural Topology Optimization Method

Structural topology optimization is born to solve the material distribution. It can find the best material distribution under the given special conditions, so that each part meets the load and boundary conditions. Moreover, letting the entire structure of the material to play the maximum performance, to achieve structural lightweight. The birth of structural topology optimization first came from Michell [2] in 1904, who proposed the lightest optimal design criterion for truss structures under single conditions and stress constraints. In 1960, Schmit [3] used mathematical programming methods to solve elastic structure design under various loads, which made structural optimization enter a new era. The mathematical induction method of structural optimization has been applied and developed rapidly. However, this actual structure also has the characteristics of low design efficiency, poor economy, and difficult to popularize [4]. Michell theory has also had important developments in recent decades. Cox [5] proved the truss with minimum flexibility design. Rozvany and Zhou [6] put forward the continuum-based optimality criteria (COC) algorithm by combining the COC theory with the finite element method. This method has the advantages of high computational efficiency and large solution scale, but it is not widely applicable [4]. So far, it is still very difficult to solve Michell theory, and its optimization results are not trusses in the engineering sense, but heterogeneous truss-like structures, which are not easy to be directly applied in engineering practice [1].

2.1. Homogenization Method

Bendsoe et al. first proposed the concept of homogenization method in 1988 [7]. Homogenization theory is a method that introduces the definition of microporous structure, represents the macroscopic characteristic parameters of materials, and reflects the changes of microscopic parameters with the amount of macroscopic parameters [8]. Through iterative calculation, the element structure is changed by adding or deleting, and a structure with higher material utilization rate is obtained. This method does not need to specifically describe the material properties of each point, but uses a characterization method to express the material properties of the microstructure [9]. On this basis, Guedes and Kikuchi [10] carried out a study on the topology optimization of two-dimensional and three-dimensional continuum structures. Hassani and Hinton [11, 12] established a homogenization equation with strong applicability. A new mathematical model of topology optimization is proposed. The corresponding optimization results are calculated. This method is summarized from the theoretical level [9]. When it comes to the global optimal problem, the correctness of the optimal solution cannot be guaranteed by this calculation method. There are problems of numerical instability and relatively low computational efficiency in the solving process [9].

2.2. Variable density method

The homogenization method still has some defects. Solid Isotropic Material with Penalization (SIMP), Rational Approximation of Material Properties (RAMP) and other methods have been proposed to form a complete theoretical system [13-15]. The structural design domain is divided into a finite number of units. Define the density of each cell to describe the overall structure. Define densities 1 and 0 as the material and material free parts of the structure. By applying density penalty mechanism to define the relationship between material density and Young's modulus, the result is closer to the 0/1 distribution. The SIMP interpolation model is the most widely used, as shown in Eq. 1.

$$E_e = \rho_e^p E_0 \quad (1)$$

where E_e and ρ_e represent the Young's modulus and density of the material respectively, p is the penalty coefficient, E_0 is the Young's modulus when the material density is 1.

Variable density method has significant advantages, such as clear definition, simple process, efficient and stable characteristics. It is widely used in topological optimization to solve many kinds of

mechanical physical fields. The goal is as a simple method to reduce the complexity of the homogenization method and improve the convergence of 0-1.

2.3. Sequential Linear Programming Method

Over time, it was discovered that the variable density method was computationally expensive and complex. In contrast, the calculation of the progressive structure method is more flexible and efficient. In 1993, Xie and Steven [16] divided the structure into many units, set a variable for each unit, and then analyzed the help of each unit to the structure. After analysis, it can be found that the stress in the structure is not evenly distributed. Parts with high stress will destroy the structure, while parts with low stress will not be able to function. Therefore, the parts with low stress can be removed after analysis to make the structure lighter, and the best topology and shape can be obtained using the removal criterion [17]. They call this approach the “Sequential Linear Programming.” However, this early approach was not complete from the start, and it had many problems. Querin [18] later discovered that this method cannot be restored if the structure is directly removed from the beginning. He studied this problem and invented a method called Bi-directional Evolutionary Structural Optimization (BESO), which allows the deleted parts to be included in the structure. In 2009, Huang and Xie [19] improved the accuracy of unit sensitivity, which is a new BESO technology. The improvement of this method has allowed this algorithm to find the best solution effectively and quickly. Now the progressive structure method is to update and iterate round by round, and each update is one step closer to the optimal solution. This method has high computational efficiency. It linearizes nonlinear problems, so the calculation is simple and efficient. However, this method still has shortcomings and cannot efficiently solve these problems when encountering some unique constraints or complex models.

2.4. Independent Continuous Mapping Method

The independent continuous mapping method is more detailed in material distribution than other methods. In 1998, in order to unify topological variables, Sui [20] used the independent continuous mapping method to turn variables such as cross section, width, density, etc. into independent variables. This method can solve some discrete topology optimization models. Convert these discrete models into continuous models, then define independent topological variables and use functions to filter and delete unnecessary variables [21]. The continuous obtained after optimization is mapped back to the discrete design variable space. The advantage of this method is that it can convert discrete problems into continuous problems, and then use mature and stable continuous optimization technology to solve the problem efficiently. However, this method also has limitations. There are some errors when mapping discrete design variables to continuous variables, which may affect the design accuracy.

2.5. Level Set Method

Compared with homogenization method, variable density method and asymptotic structure optimization method, which adopt physical description, level set method has unique advantages in boundary processing [22]. The level set method was first proposed by Osher and Sethian in 1988 [23]. In 2000, Sethian first applied it to topology optimization and used stress as a function variable to drive the evolution of the level set velocity field. The full load design of continuum structure shape topology is realized [22]. Since then, the level set method has undergone many changes in recent years. Bubble method proposed by Eschenauer et al. [24]. The H-J equation method and the semi-Lagrange method were solved by semi-implicit addition [25]. The key feature of all level set methods is to represent the boundary as the zero level set of an implicit function. In the optimization based on level set, the sensitivity update implicit function is used to optimize the position of the boundary [25].

3. Comparison and Application Analysis of Topology Optimization Methods

The above is the homogenization method, variable density method, progressive structure method, independent continuous mapping method, level set method of these methods development process,

optimization principle. Table 1 compares these five structural topology optimization methods from the perspectives of optimization objects, optimization results, application examples, and limitations.

Table 1. Comparison of structural topology optimization methods

Method	Optimization objective	Optimization effect	Practical application	Limitations
Homogenization method	Minimum flexibility	Demonstrates material distribution on a macro scale; improving overall performance	Composite structures; Porous structure; building materials, etc.	(1) The mathematical model is complex and the amount of calculation is large. (2) Accuracy relies on the assumption of homogenization and the precise description of the microstructure. (3) Assumptions may not fully correspond to actual material behavior.
Variable density method	Minimum flexibility	Material minimization while ensuring excellent performance	Car bodies; Bridges; 3D printed structures	(1) The results depend on the division of the grid, which will increase the computational cost. (2) The optimization results may not meet the required smooth transition or continuous characteristics in some applications.
4. Sequential Linear Programming	Multiple target	Optimizing thermal conductivity performance; Reducing weight; Improving vibration characteristics; reducing noise	Airplane wing; Automobile body structure	(1) Scale adjustment is required. (2) The optimization results may not be fully applicable to actual production.

5. Independent Continuous Mapping	Multiple target	Transform structures into continuous material distribution and shapes; optimizing fluid flow	Satellite structure; Prostheses in biomedical engineering; Medical devices	(1) The calculation is complex and requires a lot of computing resources and time, (2) Higher expertise and technical support.
6. Level set method	Multiple target	Generating optimization results with smooth boundaries and continuous interfaces;	Aircraft parts; 3D printed parts; customized prostheses and implants; suitable for processing complex geometries	(1) The numerical representation of the level set function may be irregular and needs to be reinitialized regularly. (2) There are difficulties with fine structures and interface energies.

It can be seen from Table 1 that homogenization method and variable density method are similar in optimization objectives. In practical application, homogenization method is more inclined to building composite materials, while variable density method is more accurate in complex structure optimization. It is difficult to apply complex computation and optimized structure. The progressive structure method has a high demand for scale optimization and is applied to the optimization of complex structural components. The independent continuous mapping method has been widely used in fluid flow and bioengineering. The level set rule is more suitable for boundary and surface problems.

7. Conclusion

This paper studies the topology optimization method of structure and its application, and obtains the following main conclusions.

(1) The homogenization method is suitable for dealing with the macroscopic properties of microstructure materials, but it still has shortcomings in the global optimal solution problem. The variable density method is widely used because of its high efficiency and stability, but the computational cost is high. The Sequential Linear Programming method is computationally flexible and efficient, but performs poorly when dealing with complex models. The independent continuous mapping method can effectively solve discrete problems, but there are errors in design accuracy. The level set method has unique advantages in boundary processing, but has high requirements on computing resources and computing time. Topology optimization methods have broad application prospects in structural lightweight and high-performance design, but these methods still have problems.

(2) The topology optimization method should be more realistic, and the optimization of structural boundaries tends to be smoother. The links between different methods are also becoming closer as they develop. Homogenization method and variable density method are all based on density discussion. The progressive structure method, variable density method, and independent continuous mapping method all use microstructure and material punishment mechanisms for optimization. The level set rule pays more attention to boundary optimization. In the development of methods in recent years, the boundary between the level set method and the density-based topology optimization

method is not so obvious. The development of a global object-oriented method is of great significance to future development.

(3) With the development of industrial manufacturing levels, artificial composite materials have become possible, and topology optimization will have broad prospects in the fields of microstructure, artificial intelligence, and aerospace.

Authors Contribution

All the authors contributed equally and their names were listed in alphabetical order.

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